DECENTRALIZED OPTIMIZATION ON TIME-VARYING DIRECTED GRAPHS UNDER COMMUNICATION CONSTRAINTS

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BACKGROUND

- Decentralized optimization problems: all clients in the network to collaboratively learn the model via communication

- Potential issues on privacy, unreliable communication and resource constraint

PROBLEM FORMULATION

- Decentralized problems over directed and time-varying networks:

$$\min_{\mathbf{x}\in\mathbb{R}^d}\left[f(\mathbf{x}) := \frac{1}{n}\sum_{i=1}^{n} f_i(\mathbf{x})\right]$$

- The agents collaborate to solve the problem by exchanging information over a network
- The network is modeled by a time-varying directed graph, $\mathcal{G}(t) = (|n|, \mathcal{E}(t))$
- The exchanged information is compressed before communication

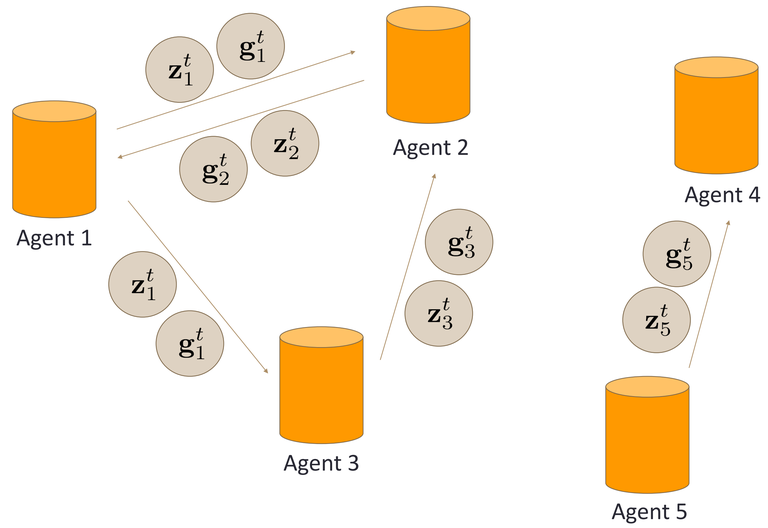
CHALLENGES

- Our algorithm is a communication-compression algorithm over time-varying directed network:
  - Communication imbalance in directed time-varying networks
  - Bias induced by the compression operator

- Compression operator: a sparsification operator that uniformly select k out of d entries from a d-dimensional message

$$Q : \mathbb{R}^d \to \mathbb{R}^d$$

ALGORITHM



$\mathbf{z}_i^t$ → Model-related vector

$\mathbf{g}_i^t$ → Gradient vector

$$\mathbf{z}_i^t = \begin{cases} \mathbf{x}_i^t, & i \in \{1, ..., n\} \\ \mathbf{y}_{i-n}^t, & i \in \{n+1, ..., 2n\} \end{cases}$$

$\mathbf{x}_i^t$ Vector of parameters

$\mathbf{y}_i^t$ Auxiliary vector to record the network imbalance bias

- Elementwise update:

$$z_{im}^{t+1} = \sum_{j=1}^{2n} [\bar{M}_m^t]_{ij} [Q(\mathbf{z}_j^t)]_m + \mathbf{1}_{\{t \bmod \mathcal{B}=\mathcal{B}-1\}} \epsilon [F]_{ij} z_{jm}^{\mathcal{B}\lfloor t/\mathcal{B}\rfloor} - \mathbf{1}_{\{t \bmod \mathcal{B}=\mathcal{B}-1\}} \alpha_{\lfloor t/\mathcal{B}\rfloor} g_{im}^{\mathcal{B}\lfloor t/\mathcal{B}\rfloor}$$

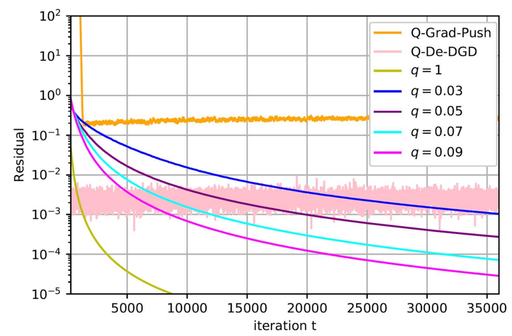
- Reweighting mixing matrix to cancel out compression bias
- Stored message facilitating consensus convergence in jointly-connected networks
- Display local gradient descent in jointly-connected network

CONVERGENCE GUARANTEE

For the stepsize $\alpha_t = \mathcal{O}(1/\sqrt{t})$, the algorithm attains the convergence rate $\mathcal{O}(\frac{\ln T}{\sqrt{T}})$.

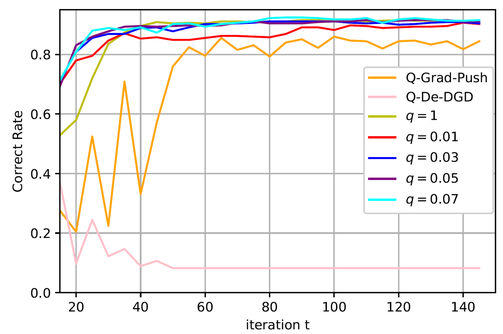
SIMULATION RESULTS

- Decentralized linear regression simulation:



Algorithm with different sparsification levels

- Decentralized logistic regression simulation:



Algorithm with different sparsification levels

CONCLUSION AND FUTURE WORK

- Proposed a communication-sparsifying algorithm for decentralized convex optimization over directed time-varying graphs
- Proved the convergence rate of the proposed algorithm
- Justified the performance of the algorithm in simulations

Future work

- Study other gradient methods to accelerate convergence rate
- Extend the algorithm to non-convex optimization problems