Online Learning with Implicit Exploration in Episodic Markov Decision Processes

Mahsa Ghasemi, Abolfazl Hashemi, Haris Vikalo, Ufuk Topcu

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Sequential Decision Making

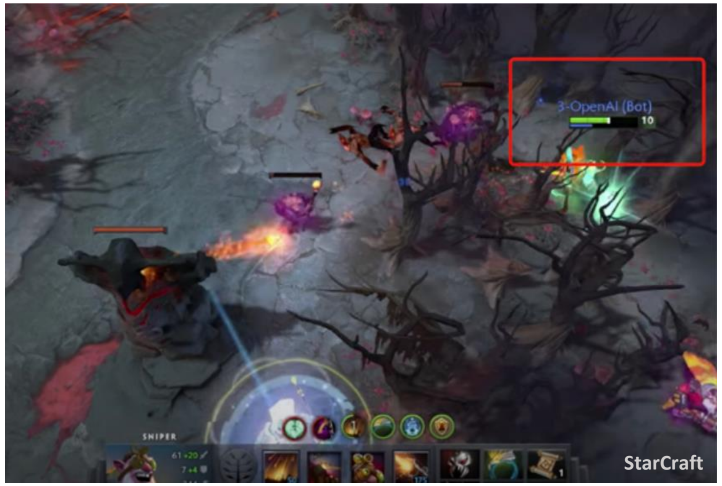
DeepMind

Sequential Interaction with the environment

Learning from a fixed reward

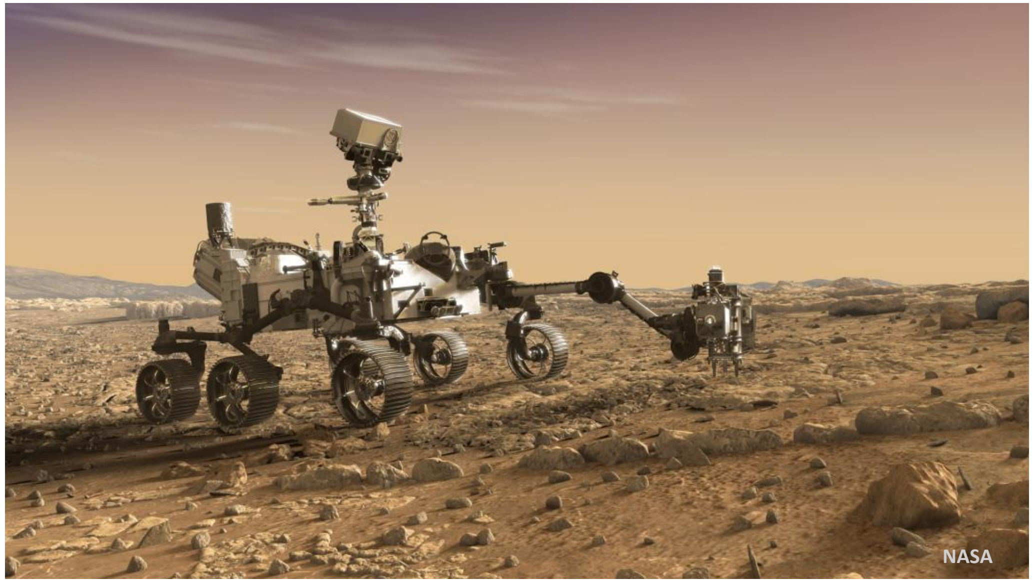
Offline: access to a lot of data

Tesla


StarCraft

KUKA

Sequential Decision Making with Varying Tasks



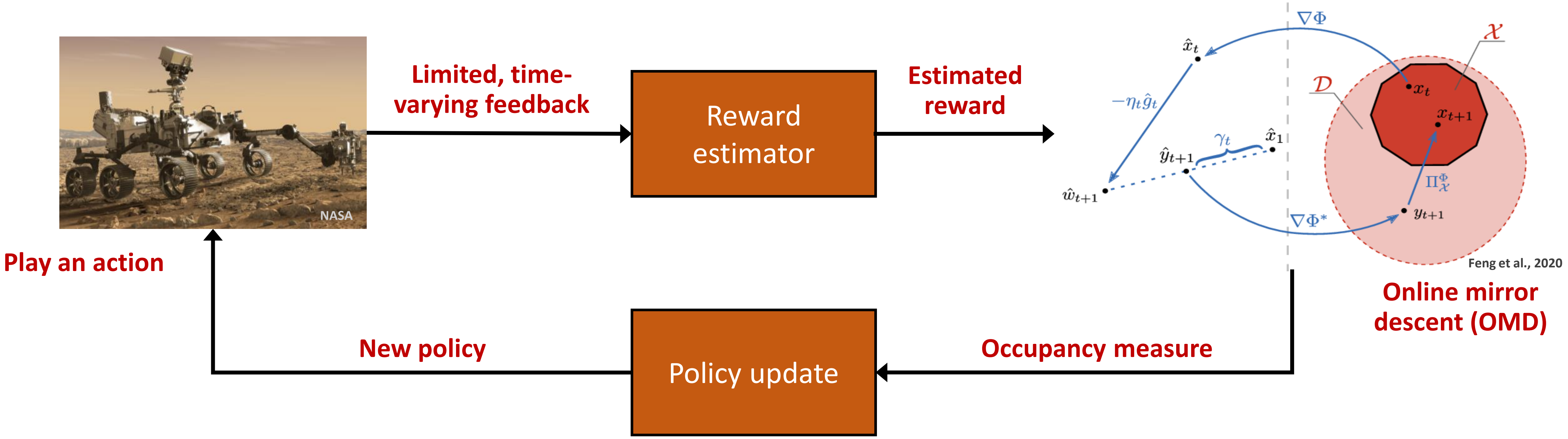
Evolving environment and task

Safety-critical operation

Limited feedback from the environment

How can we design online algorithms with high probability guarantees for varying tasks?

Online Learning with Implicit Exploration for Varying Tasks



Contributions:

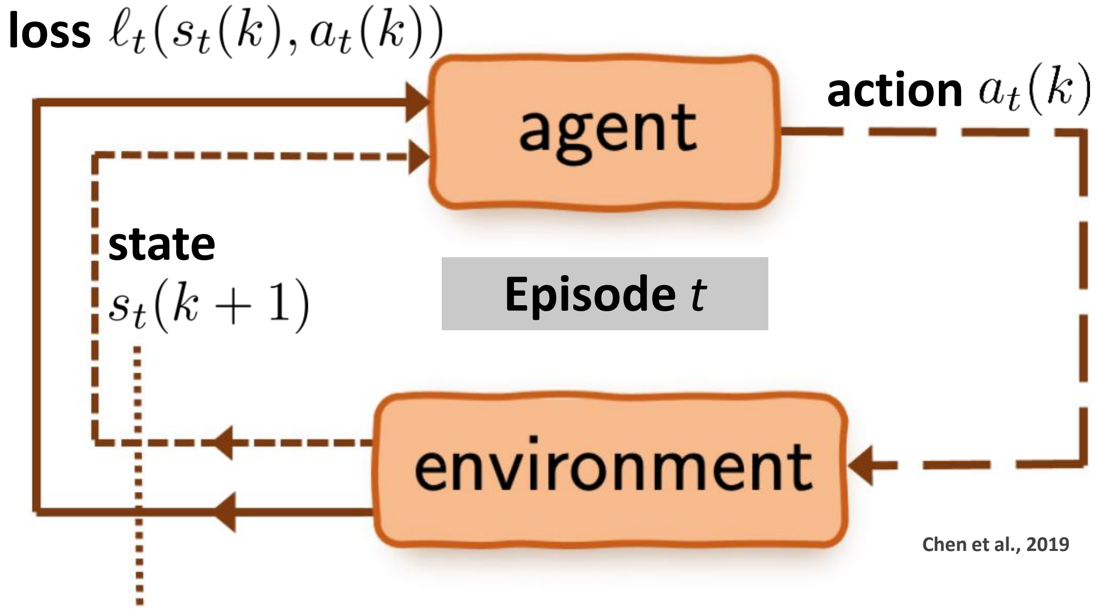
- A novel optimistically-biased reward estimator for implicit exploration
- Policy search using online mirror descent (OMD)
- Minimax optimal regret bound with high probability

Adversarial Markov Decision Process (A-MDP)

$$\mathcal{M} = (\mathcal{S}, \mathcal{A}, \mathcal{P}, \ell_t)$$

- $\mathcal{S}$: finite state space
- $\mathcal{A}$: finite action space
- $\mathcal{P}$: probabilistic transition function
- ℓ_t: time-varying loss function

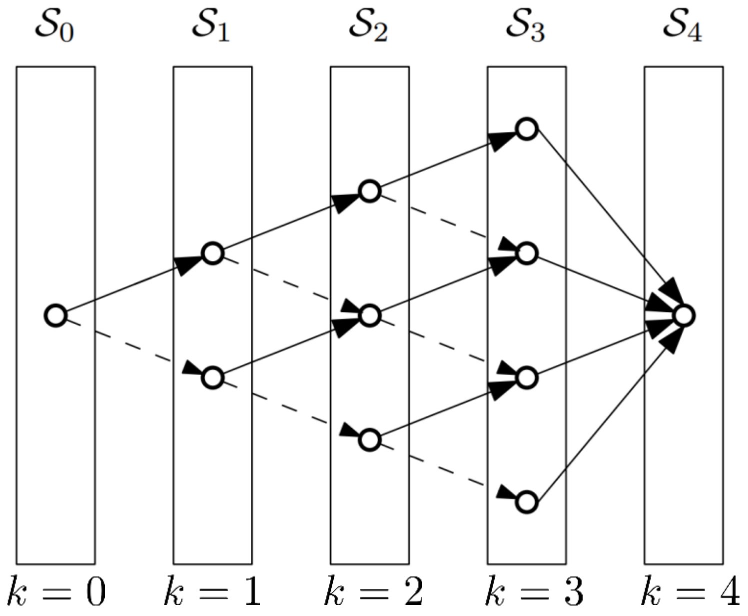
Bandit feedback



Chen et al., 2019

Loop-free episodic A-MDP:

- States are partitioned into layers
- Transition only exists from one layer to the next



Neu et al., 2020

Agent's Policy Representation via Occupancy Measure

Looking for a time-varying stochastic policy $\pi_t : \mathcal{S} \times \mathcal{A} \rightarrow [0, 1]$

Occupancy measure: the probability induced over state-action pairs by executing a policy

$$\rho^{\pi}(s, a) = \Pr(\mathbf{s}_{k(s)} = s, \mathbf{a}_{k(s)} = a | \pi)$$

Stochastic stationary policy given an occupancy measure

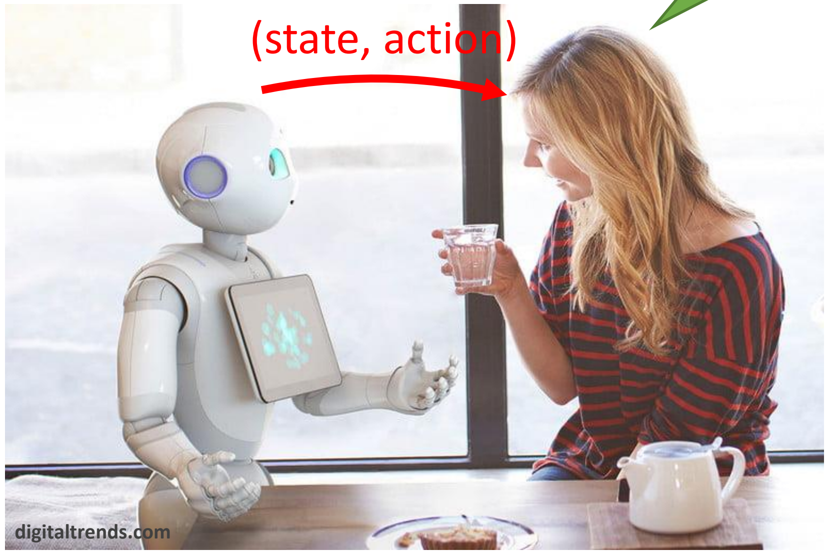
$$\pi^{\rho}(a|s) = \frac{\rho(s, a)}{\sum_{a' \in \mathcal{A}} \rho(s, a')} , \quad \forall (s, a) \in \mathcal{S} \times \mathcal{A}$$

Regret Minimization

episode *t*

task *t*

(state, action)



(loss)

Unknown and time-varying loss function (A-MDP)

$$\mathcal{M} = (\mathcal{S}, \mathcal{A}, \mathcal{P}, \ell_t)$$

Bandit feedback

$$\ell_t(s_t(k), a_t(k))$$

Learn a policy with sublinear regret:

$$\mathcal{R}_T := \max_{\pi} \mathcal{L}_T - \mathcal{L}_T(\pi)$$

best fixed policy in hindsight

Question: Can we obtain low regret with high probability?

Optimistic Loss Estimator

Bandit feedback → Estimating the loss of all state-action pairs

Goal: Obtain a low-variance loss estimator

A novel optimistically biased estimator for the loss function:

$$\hat{\boldsymbol{\ell}}_t(s,a) = \frac{\ell_t(s,a)}{\boldsymbol{\rho}_t(s,a)+\gamma}\mathbb{I}\{(s,a) \in \mathbf{h}(t)\}$$

history at current episode

exploration parameter

Optimistically biased

$$\mathbb{E}\left[\hat{\boldsymbol{\ell}}_t(s,a) | \mathbf{h}(t-1)\right] \leq \ell_t(s,a)$$

→ Implicit exploration

Policy Optimization via Online Mirror Descent

Goal: Compute a new policy from the estimated loss function

An OMD algorithm utilizing the proposed loss estimator:

$$\boldsymbol{\rho}_{t+1} = \arg \min_{\rho \in \Delta(\mathcal{M})} \left\{ \underbrace{\eta \langle \rho, \hat{\boldsymbol{\ell}}_t \rangle}_{\text{loss}} + \underbrace{D(\rho \| \boldsymbol{\rho}_t)}_{\text{policy change}} \right\}$$

learning rate (η); unnormalized KL divergence (D)

Constrained optimization → Two-step procedure

$$\tilde{\boldsymbol{\rho}}_{t+1} = \arg \min_{\rho} \left\{ \eta \langle \rho, \hat{\boldsymbol{\ell}}_t \rangle + D(\rho \| \boldsymbol{\rho}_t) \right\}$$

$$\boldsymbol{\rho}_{t+1} = \arg \min_{\rho \in \Delta(\mathcal{M})} \{ D(\rho \| \tilde{\boldsymbol{\rho}}_{t+1}) \}$$

No-Regret Learning with High-Probability

Result: Establishing sublinear regret bounds both on expectation and with high-probability

Theorem: (high-probability regret bound)

Let $\delta \in (0,1)$. If

$$\eta = \gamma = \sqrt{L\frac{\log(|\mathcal{S}||\mathcal{A}|/L)}{2T|\mathcal{S}||\mathcal{A}|}},$$

with probability at least $1-\delta$,

$$\text{regret} \leq \mathcal{O}(\sqrt{LT|\mathcal{A}||\mathcal{S}|\log(|\mathcal{S}||\mathcal{A}|/L)}\log\tfrac{1}{\delta}).$$

episode length number of episodes number of states number of actions

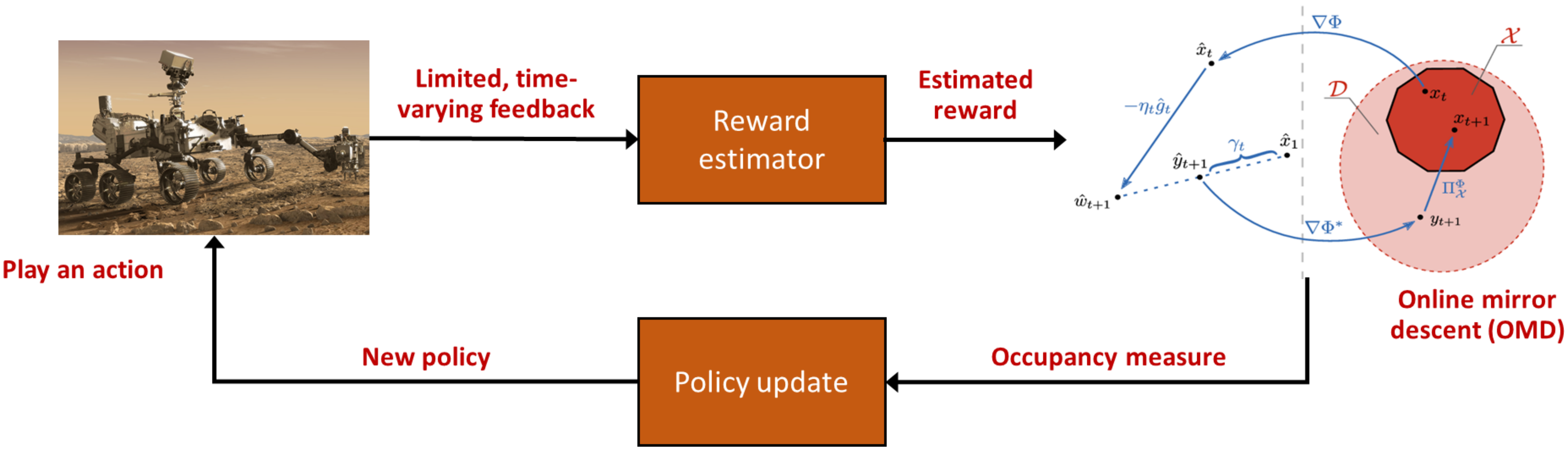
Minimax optimal regret (up to logarithmic terms)

Conclusion and Future Work

- Proposed an optimistic loss estimator for learning in episodic A-MDP under bandit feedback
- Developed an OMD policy optimization utilizing the proposed loss estimator
- Established a minimax optimal regret bound with high probability

Future Directions

- Parameter-free and anytime algorithms
- Unknown, time-varying dynamics and large-scale state spaces



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